1. (20%) 

It is known that $AB = I$, and $A = LDL^t$, where $A$ and $B$ are $5 \times 5$ matrices, $I$ is a $5 \times 5$ unit matrix, $L^t$ is the transpose of matrix $L$. It is known that:

$$
L = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & -2 & 1 & 0 & 0 \\
0 & -3 & 0 & 1 & 0 \\
0 & -4 & 0 & 0 & 1
\end{bmatrix}, \quad D = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 \\
0 & 0 & 4 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}.
$$

Please determine the inverse of $L$ and the values of entries $B_{2j} \ (j = 1-5)$ of matrix $B$.

2. (20%) 

Let $F(x)$ be the function defined by

$$
F(x) = \sum_{n=\infty}^{\infty} \cosh(x - 2nL)[u(x - (2n - 1)L) - u(x - (2n + 1)L)]
$$

where $L$ is a constant, $u$ is the unit step function defined by

$$
u(x - a) = \begin{cases} 
0 & \text{for } x < a \\
1 & \text{for } x > a
\end{cases}
$$

(a) Determine $F'(x)$, the derivative of $F(x)$ with respect to $x$.

(b) Sketch the graph of $F(x)$ and $F'(x)$.

(c) Find the Fourier series of $F(x)$

3. (20%) 

Evaluate $\iint xdydz + ydzdx + zdx dy$ over the surface $z = 1 - x^2 - y^2$ for $x^2 + y^2 \leq 1$, oriented by the upper normal.
4. (20%) Please solve the initial value problem
\[ y''' - 2y'' - y' + 2y = 2x^2 - 6x + 4 \]
\[ y(0) = 5 \quad y'(0) = -5 \quad y''(0) = 1 \]

5. (20%) A system of second-order differential equation is given by
\[
\begin{bmatrix}
1 & 0 \\
0 & 2
\end{bmatrix}
\begin{bmatrix}
y_1'' \\
y_2''
\end{bmatrix}
+ \begin{bmatrix}
2 & -1 \\
-1 & 3
\end{bmatrix}
\begin{bmatrix}
y_1' \\
y_2'
\end{bmatrix}
= \begin{bmatrix}
0 \\
 f(t)
\end{bmatrix}.
\]

(a) Assume that the solution is of the form
\[
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix}
= \begin{bmatrix}
y_1(t) \\
y_2(t)
\end{bmatrix}
= \begin{bmatrix}
Y_1 \\
Y_2
\end{bmatrix}e^{\omega t}
\]
if \( f(t) = 0 \). Find the solution
\[
\begin{bmatrix}
y_1(t) \\
y_2(t)
\end{bmatrix}
\]
satisfying the initial conditions
\[
\begin{bmatrix}
y_1(0) \\
y_2(0)
\end{bmatrix}
= \begin{bmatrix}
1 \\
3
\end{bmatrix}, \quad \begin{bmatrix}
y_1'(0) \\
y_2'(0)
\end{bmatrix}
= \begin{bmatrix}
-\sqrt{10} \\
0
\end{bmatrix}.
\]

(b) If \( f(t) = \sin(\omega t) \), determine the range of \( \omega \) such that the particular solution
\[
\begin{bmatrix}
y_{1p}(t) \\
y_{2p}(t)
\end{bmatrix}
\]
are in the same phase, i.e., both \( y_{1p}(t) \) and \( y_{2p}(t) \) have the same sign.